CGI Professional Development
University of California, Los Angeles

Summer 2014

Presented by: Melissa Canham
Getting to Know You

- Mix-Freeze-Group
- Team Interview
- Quadrant Partners
- 3 Interesting Things
- One of These Things
Mix-Freeze-Group

1. Music starts-Mix around the room and greet all people you see
2. Freeze when music stops
3. Facilitator will ask a question
4. Participants show answer on fingers
5. Participants group according to answer
6. Form groups and prepare to engage
Quadrant Partners

- Quadrant I: Find 2 people who teach a similar grade level as you
- Quadrant II: Find 1 person who enjoys a similar hobby as you
- Quadrant III: Find 1 person who teaches a different grade level than you
- Quadrant IV: List the 3 people you met during the team interview
One of These Things
<table>
<thead>
<tr>
<th>9</th>
<th>8</th>
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<td>2</td>
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<td>400</td>
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</table>
One of These Things: Debrief

- Where’s the math?
- Development of a critical eye
  - What possible benefits do you see for students to engage in this type of task?
  - When might this be a good task to pose to students?
  - How do we help students think about what they are doing (meta-cognition) and provide structure to developing these skills?
Today’s Agenda

- Common Core Overview
- Introduction to CGI
  - Research Problems
  - Problem Types
- Single Digit Strategy Development
- Algebraic Reasoning: Equal Sign
- Reflection
Day 2 Agenda

- Math Game: Double Compare
- Multi-Digit Strategies
- Unpacking Problems
- Choral Counting
- Relational Thinking: Number Strings
- CGI & the Common Core
- Reflection
Day 3 Agenda

- Grouping Problems: Multiplication & Division
- Classroom Connection: Analyzing Student Work
- Fair Share & Fractions
- Counting Collections
- Big Ideas Review
- Planning: CGI in Your Classroom
- Reflection
Day 4 Agenda

- Tasks to Develop Number Sense
- Looking for Problem Solving in the CCSS-M
- Responsiveness in the Classroom
- Classroom Connection: Number Strings
- Choral Counting
- Big Ideas Review
- Planning: CGI in Your Classroom
A Look Inside the CCSS for Mathematics
<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1.</td>
<td>Make sense of problems and persevere in solving them.</td>
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<td>Reason abstractly and quantitatively.</td>
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<td>3.</td>
<td>Construct viable arguments and critique the reasoning of others.</td>
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<td>4.</td>
<td>Model with mathematics.</td>
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<td>5.</td>
<td>Use appropriate tools strategically.</td>
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<td>6.</td>
<td>Attend to precision</td>
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<td>7.</td>
<td>Look for and make use of structure</td>
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<td>8.</td>
<td>Look for and express regularity in repeated reasoning.</td>
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## CCSS Domain Progression

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David Coleman
Three Shifts of the Common Core in Mathematics

- **Focus** strongly where the standards focus
- **Coherence**: Think across grades and link to major topics within grades
- **Rigor**: In major topics, pursue:
  - Conceptual understanding,
  - Procedural skill and fluency, and
  - Application

with equal intensity
<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
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</thead>
<tbody>
<tr>
<td>K-2</td>
<td><strong>Addition and subtraction</strong>&lt;br&gt;concepts, skills, and problem solving, and place value</td>
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<tr>
<td>3-5</td>
<td><strong>Multiplication and division of whole numbers and fractions</strong>&lt;br&gt;concepts, skills, and problem solving</td>
</tr>
<tr>
<td>6</td>
<td><strong>Ratios and proportional reasoning; early expressions and equations</strong></td>
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<tr>
<td>7</td>
<td><strong>Ratios and proportional reasoning; arithmetic of rational numbers</strong></td>
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<tr>
<td>8</td>
<td><strong>Linear algebra and linear functions</strong></td>
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</table>
## Required Fluencies in K-6

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluency</th>
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<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
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<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20</td>
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<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
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<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Multiply/divide within 100</td>
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<tr>
<td></td>
<td>3.NBT.2</td>
<td>Add/subtract within 1000</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Add/subtract within 1,000,000</td>
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<td>5</td>
<td>5.NBT.5</td>
<td>Multi-digit multiplication</td>
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<tr>
<td>6</td>
<td>6.NS.2,3</td>
<td>Multi-digit division</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multi-digit decimal operations</td>
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</tbody>
</table>
What is meant by fluent?

- **Efficient** – a reasonable amount of steps

- **Accurate** – the solution has a degree of precision appropriate for the problem context. The student knows why the solution works.

- **Flexible** – the student knows several strategies
  - 9 x 6 is 54 because…
    - 6 x 10 is 60, take away one group of 6 and it is 54
    - 5 x 6 is 30, 4 x 6 is 24, 30 + 24 is 54
    - 9 x 3 is 27, double that 20 + 20 is 40, 7 + 7 is 14, 40 + 14 is 54
Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas:

1. Extending understanding of base-ten notation;
2. Building fluency with addition and subtraction;
3. Using standard units of measure; and
4. Describing and analyzing shapes.

1) Students extend their understanding of the base-ten system, which includes ideas of counting in fives, tens, and multiples of hundreds, thousands, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000.
Grade Level Overview

Grade 2 Overview

Operations and Algebraic Thinking
- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data
- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry
- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Format of K-8 Standards

Operations and Algebraic Thinking

2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.¹

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.² By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Research Items

Directions: Rank these problems from easiest to most difficult.

A. Carla has 7 dollars. How many more dollars does she have to earn so that she will have 11 dollars to buy a puppy?

B. Mr. Gomez had 20 cupcakes. He put the cupcakes into 4 boxes so that there were the same number of cupcakes in each box. How many cupcakes did Mr. Gomez put in each box?

C. Paco had 13 cookies. He ate 6 of them. How many cookies does Paco have left?

D. Tad had 15 guppies. He put 3 guppies in each jar. How many jars did Tad put guppies in?

E. Robin has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Robin have altogether?

F. Hannah has 12 balloons. Jacob has 7 balloons. How many more balloons does Hannah have than Jacob?
## Kindergarten Problems

- Work through these items, trying to identify ways that young children might solve these problems.
- Note strategies you used to share later.
- Individually rank the problems from easiest (1) to most difficult (6) to solve.
- Rank as a group – come to a consensus on disagreements.
<table>
<thead>
<tr>
<th>A. Carla has 7 dollars. How many more dollars does she have to earn so that she will have 11 dollars to buy a puppy?</th>
<th>80%</th>
<th>Ranking: 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>B. Mr. Gomez had 20 cupcakes. He put the cupcakes into 4 boxes so that there were the same number of cupcakes in each box. How many cupcakes did Mr. Gomez put in each box?</td>
<td>70%</td>
<td>Ranking: 6</td>
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<tr>
<td>C. Paco had 13 cookies. He ate 6 of them. How many cookies does Paco have left?</td>
<td>88.6%</td>
<td>Ranking: 1</td>
</tr>
<tr>
<td>D. Tad had 15 guppies. He put 3 guppies in each jar. How many jars did Tad put guppies in?</td>
<td>74.3%</td>
<td>Ranking: 4</td>
</tr>
<tr>
<td>E. Robin has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Robin have altogether?</td>
<td>87.1%</td>
<td>Ranking: 2</td>
</tr>
<tr>
<td>F. Hannah has 12 balloons. Jacob has 7 balloons. How many more balloons does Hannah have than Jacob?</td>
<td>72.9%</td>
<td>Ranking: 5</td>
</tr>
</tbody>
</table>
Dialogue

- Take a moment to talk to someone next to you about what you noticed about the difficulty of the problems and the rankings.

- Be prepared to share one of your ideas.
Video

- While watching the video, pay attention to:
  - Student strategies for solving
  - What might you do as a teacher to help this student develop more sophisticated or efficient problem solving strategies?

- Talk to a partner about what you noticed in the video. Did anything surprise you? What questions might you have?
Revisiting the Research Items

- Translate each story problem into a number sentence or equation.
  - What do you notice about the position of the unknown quantity in each equation?
  - How does the position of the unknown quantity appear to affect the difficulty in the problem?
Classification of Word Problems

- Not all addition and subtraction problems are the same. What are some of the distinguishing factors that relate to the difficulty in solving various problems?
  - Location of the unknown variable
  - The types of actions or relationships described in the problem
Your Task

Collaboration time: Work with a colleague from a similar grade level to design a **Join-Change-Unknown** problem to pose to your students that has a relevant context and numbers that make sense for your grade level.
Reflection on the Morning

- What does a good problem solver look like?
- What more do you know about CGI?
- What are the big ideas?
- What questions do you have?
Three Interesting Things

- Everyone, please stand up.

- The facilitator will read the statements on a given card.

- If the statement pertains to you or is true for you, stay standing. If the statement is not true for you, sit down.
Children’s Solution Strategies

- Direct Modeling
- Counting Strategies
- Derived Facts
- Recall/Number Facts
Oliver’s Toy Cars

- Oliver has 5 toy cars. His sister gives him 8 more toy cars for his birthday. How many toy cars does Oliver have now?
- Please solve this problem in at least two different ways.
- Compare your solution strategies with colleagues at your table.
Can anyone tell me a story that corresponds to this number sentence?

7 + ___ = 15

Solve this problem in at least two different ways. Write one solution on one post-it note. Use a different post-it note for each solution.
Sorting Problem Solving Strategies

- Work with the people at your table to group your solution strategies along the problem solving trajectory.
  - Which solutions fall under what categories?
  - What are some similarities and differences amongst the solution strategies you generated?
  - What questions might this raise for you?
Problem Solving Trajectory

- In looking at multiple solution strategies from this perspective,
  - What has this caused you to think about?
  - How might you be able to use what we have talked about to help guide decision-making in your classroom?
  - What questions might you have?
Algebraic Reasoning: Equal Sign

- $8 + 4 = \_\_ + 5$

- What do you think students will say goes in the blank?

- What do kids understand about the equal sign based on the answer they give?
### Student Responses

$$8 + 4 = \_\_ + 5$$

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<thead>
<tr>
<th>Grade</th>
<th>7</th>
<th>12</th>
<th>17</th>
<th>12 and 17</th>
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<tbody>
<tr>
<td>1 and 2</td>
<td>5</td>
<td>58</td>
<td>13</td>
<td>8</td>
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<tr>
<td>3 and 4</td>
<td>9</td>
<td>49</td>
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<td>10</td>
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<td>5 and 6</td>
<td>2</td>
<td>76</td>
<td>21</td>
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</table>
True/ False
Number Sentences

• Determine if the each number sentence is true or false.

• Provide a reason or justification for your answer. What do you think students might say?
The goal of the teacher’s work in this video is to push Spencer’s ideas about the equal sign.

- As you watch this video, pay attention to Spencer’s thinking.
- What do you think Spencer believes about the equal sign?
- What are some teacher moves that may have pushed Spencer’s thinking?
Next Steps.....

• What has this session caused you to think about?

• Efforts to foster algebraic reasoning and mathematical understanding of the equal sign…
  • What can be done at each grade level to help students understand the equal sign?
  • What might you do as a teacher towards fostering algebraic reasoning and understanding of the equal sign?
  • What can we do as a school or district through teacher collaboration towards this effort?
Children’s Mathematics

- Read Chapters 1, 2 & 3 to review today’s learning

- Have a great evening and we will see you tomorrow!