Developing Place Value Understanding through Problem Solving

GLAMC Mini Conference #3

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WHERE IS PLACE VALUE IN THE CCSS?

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<td>Number and Operations in Base Ten</td>
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<td><strong>Multiplication and division of whole numbers and fractions</strong>&lt;br&gt;concepts, skills, and problem solving</td>
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<td><strong>Ratios and proportional reasoning; early expressions and equations</strong></td>
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What is the end goal?

- 4th grade Narrative from the CCSS-M:
  - Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers (CCSS-M, p.27)
Where do we start?

- K.NBT.1: Compose and decompose numbers from 11 to 19 into ten ones and some further ones, and record each composition or decomposition by a drawing or equation; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
What does Cognitively Guided Instruction (CGI) Look Like?

- Problem solving is the focus of instruction; teachers pose a variety of problems.

- Many problem-solving strategies are used to solve problems. Children decide how they should solve each problem.

- Children communicate to their teachers and peers how they solve the problems.

- Teachers understand children’s problem-solving strategies and use that knowledge to plan instruction.
Counting Collections
Counting Collections

- In groups of 2-3, take one bag of items. Your task is to count the collection of items in that bag in any way that makes sense to you.
- After you count your collection, try to record on your paper what and how you counted.
- Leave your items on your tables exactly as you counted them.
1st Grade Counting Collections -- October
STANDARDS OF MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

6. Attend to precision

7. Look for and make use of structure

8. Look for and express regularity in repeated reasoning

Overarching habits of mind of a productive mathematical thinker

- Reasoning and explaining
- Modeling and using tools
- Seeing structure and generalizing
Looking at Student Work

- What similarities and differences do you notice?
- If you have seen these collections in your classroom, what mathematics do students demonstrate understanding of in this task?
- How might you be able to help students who struggle?
Counting Collections

Names

We counted ___________
items altogether.

This is how they look:

Counting Collections

Names ___________

We counted ___________

We counted ___________
items altogether.

This is how they look:

1 2 3
1 2 3 4 5 6 7 8 9 0 1 2 3
14 15 16 15 18 19 20 21 22 23 24
25 26 27 28 29 30

---Counting Collections---

Names **Miguel & Maylinda**

We counted **pipe cleaners**

We had **52** items in our collection.

---Counting Collections---

Names **Maylinda & Miguel**

We counted **pipe cleaners**

We had **52** items in our collection.

This is how we counted our collection:
----Counting Collections----

Names  Elisha & Briana

We counted Beads

We had 48 items in our collection.

This is how we counted our collection:
Counting Collections

Names: John & Gabrie
We counted dice.
We counted 69 items altogether.

This is how they look:

Counting Collections

Names: Lindy & Karyn
We counted circles.
We counted 228 items altogether.

This is how they look:

[Diagram showing number place value with 228]
Debrief: Counting Collections

- How does this support understanding of place value?
- What possible benefits do you see for students to engage in this type of task?
- When might this be a good task to pose to students?
- How do we help students think about what they are doing (meta-cognition) and provide structure to developing these skills?
Grouping by 10 Word Problems
Groups of Ten Strategies

- Direct Modeling by Ones
- Direct Modeling by Tens
- Counting by Ones and/or Tens
- Direct Place Value
Looking at Strategies

Solve the following problem two different ways.

Our class has 7 boxes of doughnuts. There are 10 doughnuts in each box. We also have 3 extra doughnuts. How many doughnuts do we have all together?
Strategy 1:

Solution: 31

Number Sentence: 10 + 10 + 10 + 1 = 31

Explanation: I put 3 ten frames and 1 one, and I got 31.
What Strategy?

Solution: 30 - 12 = 18
Number Sentence: 10 + 10 + 10 + 1

Explanation: I use open number lines to find the answer.
What Strategy?

Our class has 7 box(es) of donuts. There are 10 donuts in each box. We also have 3 extra donuts. How many donuts do we have all together?

Solution: 73

Number Sentence: ______
What Strategy?

Our class has $\frac{17}{12}$ box(es) of donuts. There are 10 donuts in each box. We also have $\frac{4}{4}$ extra donut(s). How many donuts do we have altogether?

Strategy 1:

Solution: 124 donuts

Number Sentence: $12 \times 10 + 4 = 124$

Explanation: I used Cubes. First, I put 17 rods and 4 units. Next, I put all the 12 rods and I got 120 and then I added in the 4. I got 124.
**What Strategy?**

- Bailey’s Strategy:

  “The answer is in the number choices. For (3,1), it is 31 because you have 3 tens and 1 one. For (7, 3) it is 73. For (12, 4) the answer is 124 because you have 12 tens and 4 ones.”
Developing Understanding of Place Value

- Unitizing the ten (or hundred or thousand) in context
- Constructing meaningful solutions without instruction
- Providing many experiences with grouping by ten facilitates children’s invention of multi-digit algorithms.
Kevin Video

- How did Kevin solve the problem?
- What type of strategy did he use?
- Did Kevin’s strategy surprise you?
Number Talks

\[ 5 \times 5 = 25 \]
\[ \frac{5 + 5 + 5 + 5 + 5}{10} + 5 = \frac{25}{25} \]
\[ 5 \times 4 = 20 \]
\[ 5 \times 4 + \frac{5}{25} \]
\[ 5 \times 10 = 50 \]
\[ 5 \times 5 = 25 \]
\[ \frac{5 \times 10}{50} = \frac{100}{50} \]
\[ 5 \times 20 = 100 \]
\[ 5 \times (2 \times 10) = 10 \times 100 \]
\[ \frac{5 \times 19}{95} \]
\[ 5 \times (10 + 9) = 95 \]
\[ \frac{5 \times 10}{50} = \frac{25}{95} \]
\[ 5 \times 4 = 20 \]
What is a Number Talk?

- 5 – 15 minutes during the opening of math time
- Short lesson alongside (but not necessarily directly related to) the ongoing math curriculum
- Provide students with meaningful ongoing practice with:
  - Computation
  - Number Sense
  - Place Value
  - Standards for Mathematical Practice
  - Listening to others’ strategies
  - Properties of Operations
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- Children communicate to their teachers and peers how they solve the problems.

- Teachers understand children’s problem-solving strategies and use that knowledge to plan instruction.
CGI Website

www.dusd.net/cgi
Thank You!
mcanham@dusd.net
www.dusd.net/cgi
References

- Parrish, Sherry. *Number Talks*. Math Solutions, 2010